# JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD 

## B.Tech II Year I Semester Examinations, March - 2017

 PROBABILITY THEORY AND STOCHASTIC PROCESSES(Electronics and Communication Engineering)

## Time: 3 Hours

Max. Marks: 75
Note: This question paper contains two parts A and B.
...... Part A issompulsory which corries 25 marks: Answer all questions in Part.A... Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have $\mathrm{a}, \mathrm{b}, \mathrm{c}$ as sub questions.

## PART- A

(...1:) Distinguish between Joint probability and conditional probability.
b) Give the definition and Axioms of probability.
c) Discuss exponential distribution.
d) Find the skew for Gaussian distributed random variable.
e) Explain conditional distribution function.
f) Discuss joint Gaussian random variables:
g) Distinguish deterministic and non deterministic processes.
h) Explain the covariance matrix and its properties.
i) Give the properties of cross power density spectrum.
j) Discuss the spectral characteristics of a system function.
2.a) State and prove Baye's theorem.
b) In a hostel $60 \%$ of the students read Hindi news paper, $40 \%$ read English news paper and $20 \%$ read both Hindi and English news papers. A student is selected at random:
i) Find the probability that he reads neither Hindi nor English news papers. $\qquad$
ii) If he reads Hindi news paper, find the probability that he reads English news paper.

## OR

3.a) Define the following and give example for each of the following:
i) Discrete and continuous sample space ii) Mutually Exclusive event.
b) Two cardseate drawn from a deck of 52-card deck (the first is not replaced)
i) Given the first card is a queen, what is the probability that a second card is also queen?
ii) Given the first card is a queen, what is the probability that a second card is a 7 ?
4.a) What do you mean by probability density function? State and prove its properties.
b) Explain the Rayleigh distribution and density functions.
5.a) Find the moment generating function about the origin of the Poisson distribution.
b) Determine the moment generating function of a random variable with density function $f_{x}(x)=1 / b e^{-(x-a) / b} \cdot u(x)$ :
6.a) Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.
b) State central limit theorem for unequal distributions and explain.

## OR

7.a) Two random variables $X_{1}, X_{2}$ are related to $Y$ as $Y=\left(X_{1}^{2}+X_{2}^{2}\right)^{1 / 2}$. Find the probability density function of $Y$ in terms of joint density of $X_{1}, X_{2}$.
b) Determine the constant buich that the function
$f_{x y}(x, y)=\left\{\begin{array}{cc}3 x y ; & 0<x<1 \\ 0 ; & \text { otherwise }\end{array} \quad 0<\mathrm{y}<\mathrm{b}\right.$ is a valid joint density function.
8.a) Explain the concept of Random process and Stationary process.
b) Distinguish between Auto correlation function and cross correlation function. State the properties of cross correlation function.
[5+5]

## OR

9.a) Explain classification of random process with neat sketches.
b) Prove that the power spectrum and Autocorrelation function of the random process form a Fourier Transform pair.
10.a) State and prove any three properties of Power Spectral Density.
b) Find the output power density spectrum and output Auto correlation function for a system with $\mathrm{h}(\mathrm{t})=\mathrm{e}^{-\mathrm{t}}$, for $\mathrm{t}>0$ as input with PSD $\mathrm{h}_{0} / 2$.

## OR

11:a) Derive the relation between input and output-ACF of an LTI.system with impulse response h(t):
b) Derive the properties of Cross-Power Density function.



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# JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD <br> B.Tech II Year I Semester Examinations, March - 2017 PROBABILITY THEORY AND STOCHASTIC PROCESSES 

 (Common to ECE, ETM)
## Time: 3 Hours

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Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have $\mathrm{a}, \mathrm{b}, \mathrm{c}$ as sub questions.

PART - A
1.a) Define Random variable.
b) Write about the continuous and mixed random variables.
c) Mention the difference between the Variance and Skew.
d) Write about the Rayleigh density and distribution function.
e) Explain the equal and unequal distributions.
f) Write about linear transformations of Gaussian random variables.
g) Mention the properties covariance.
h) Show that $S_{x x}(\omega)=S_{x x}(-\omega)$.
i) State wiener-Khinchin relation.
j) Express the relationship between power spectrum and autocorrelation.

## PART - B

(50 Marks)
2.a) Discuss the mutually exclusive events with an example.
b) Define probability, set and sample spaces.

OR
3. Write the classical and axiomatic definitions of Probability and for a three digit decimal number chosen at random, find the probability that exactly K digits are greater than and equal to 5 , for $0<K<3$.
4.a) Obtain the relationship between probability and probability density function.
b) Find the moment generating function of the random variable whose moments are $\mathrm{m}_{\mathrm{r}}=(\mathrm{r}+1)!2^{\mathrm{r}}$.

## OR

5.a) Write about Chebychev's inequality and mention about its characteristic function.
b) Determine the moment generating function about origin of the Poisson distribution. [5+5]
6.a) Differentiate between the marginal distribution functions, conditional distribution functions and densities.
b) Given the transformation $\mathrm{y}=\cos \mathrm{x}$ where x be a uniformly distributed random variable in the interval $(-\pi, \pi)$. Find $f_{y}(y)$ and $E[y]$.

OR
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7. Let X be a random variable defined, Find $\mathrm{E}[3 \mathrm{X}]$ and $\mathrm{E}\left[\mathrm{X}^{2}\right]$ given the density function as

$$
f_{x}(x)=\begin{array}{cl}
(\pi / 16) \cos (\pi x / 8), & -4 \leq x \leq 4  \tag{10}\\
0, & \text { elsewhere }
\end{array}
$$

8.a) State and prove properties of cross correlation function.
b) If the PSD of $X(t)$ is $S_{x x}(\omega)$. Find the PSD of $d x(t) / d t$.

## OR

9. A random process $\mathrm{Y}(\mathrm{t})=\mathrm{X}(\mathrm{t})-\mathrm{X}(\mathrm{t}+\tau)$ is defined in terms of a process $\mathrm{X}(\mathrm{t})$. That is at least wide sense stationary.
a) Show that mean value of $Y(t)$ is 0 even if $X(t)$ has a non Zero mean value.
b) If $\mathrm{Y}(\mathrm{t})=\mathrm{X}(\mathrm{t})+\mathrm{X}(\mathrm{t}+\tau)$ find $\mathrm{E}[\mathrm{Y}(\mathrm{t})]$ and $\sigma \mathrm{Y}^{2}$.
10. The auto correlation function of a random process $\mathrm{X}(\mathrm{t})$ is $\mathrm{R}_{\mathrm{XX}}(\tau)=3+2 \exp \left(-4 \tau^{2}\right)$.
a) Evaluate the power spectrum and average power of $X(t)$.
b) Calculate the power in the frequency band $-1 / \sqrt{ } 2 \leq \omega \leq 1 / \sqrt{ } 2$

OR
11. Derive the relation between PSDs of input and output random process of an LTI system.

# JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD 

## B.Tech II Year I Semester Examinations, April/May - 2018 PROBABILITY THEORY AND STOCHASTIC PROCESSES

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Each question carries 10 Marks and may have a, b, c as sub questions.

## PART - A

1.a) Write the conditions for a function to be a random variable.
b) Explain the significance of mathematical model of experiments.
c) Write short notes on Chebychev's inequality.
d) Define Characteristic function and present generation of moments using it. [3]
e) State central limit theorem for the case of equal distributions.
f) Write the properties of jointly Gaussian random variables.
g) What is a WSS random process?
h) Write short notes on Gaussian random process.
i) Write the expression for power spectral density.
j) Write any three properties of cross-power density spectrum.

## PART - B

(50 Marks)
2. A missile can be accidentally launched if two relays A and B both have failed. The probabilities of A and B failing are known to be 0.01 and 0.03 , respectively. It is also known that B is more likely to fail (probability 0.06), if A has failed.
a) What is the probability of an accidental missile launch?
b) What is the probability that A will fail, if B has failed?
c) Are the events "A fails" and "B fails" statistically independent?

## OR

3. You (A) and two others (B and C) each toss a fair coin in a two-step gambling game. In step1 the person whose toss is not a match to either of other two is "odd man out". Only the remaining two whose coins match go on to step 2 to resolve the ultimate winner.
a) What is the probability that you will advance to step 2 after the first toss?
b) What is the probability you will be out after the first toss?
c) What is the probability that no one will be out after the first toss?
4.a) Obtain the moment generating function of a uniformly distributed random variable.
b) Obtain the variance of Raleigh random variable.
[5+5]

## OR

5.a) A random variable $X$ uniformly distributed in the interval ( $0, \pi / 2$ ). Consider the transformation $Y=\sin x$, obtain the pdf of $Y$.
b) Obtain the variance of Gaussian random variable.
6.a) The joint characteristic function of two random variables is given by $\phi_{\mathrm{XY}}\left(\omega_{1}, \omega_{2}\right)=\exp \left(-\omega_{1}{ }^{2}-4 \omega_{2}{ }^{2}\right)$. Check whether X and Y are uncorrelated or not.
b) X and Y are statistically independent random variables and $\mathrm{W}=\mathrm{X}+\mathrm{Y}$ obtain the pdf of W.

## OR

7.a) Write the properties of joint distribution function.
b) Prove that the variance of weighted sum of N random variables equals the weighted sum of all their covariances.
8. Define autocorrelation function of a random process. Write properties of auto correlation function of a WSS process and prove any three of them.
[10]

## OR

9.a) A random process $X(t)=A \cos \left(\omega_{0} t\right)+B \sin \left(\omega_{0} t\right)$ where $\omega_{0}$ is a constant and $A, B$ are uncorrelated zero mean random variables with same variances. Check whether $\mathrm{X}(\mathrm{t})$ is WSS or not?
b) Classify random processes and explain.
10. Derive the relationship between cross-power spectrum and cross-correlation function.

OR
11.a) The autocorrelation function of a random process $\mathrm{R}_{\mathrm{XX}}(\tau)=4 \cos \left(\omega_{0} \tau\right)$, where $\omega_{o}$ is a constant. Obtain its power spectral density.
b) Obtain the average power in the random process $X(t)=A \cos \left(\omega_{0} t+\theta\right)$ where $A, \omega_{o}$ are real constants and $\theta$ is a random variable uniformly distributed in the range $(0,2 \pi)$.
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# JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD 

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1.a) Write the conditions for a function to be a random variable.
b) Explain the significance of mathematical model of experiments.
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e) State central limit theorem for the case of equal distributions.
f) Write the properties of jointly Gaussian random variables.
g) What is a WSS random process?
h) Write short notes on Gaussian random process.
i) Write the expression for power spectral density.
j) Write any three properties of cross-power density spectrum.

## PART - B

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3. You (A) and two others (B and C) each toss a fair coin in a two-step gambling game. In step1 the person whose toss is not a match to either of other two is "odd man out". Only the remaining two whose coins match go on to step 2 to resolve the ultimate winner.
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b) Obtain the variance of Raleigh random variable.
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b) Obtain the variance of Gaussian random variable.
6.a) The joint characteristic function of two random variables is given by $\phi_{\mathrm{XY}}\left(\omega_{1}, \omega_{2}\right)=\exp \left(-\omega_{1}{ }^{2}-4 \omega_{2}{ }^{2}\right)$. Check whether X and Y are uncorrelated or not.
b) X and Y are statistically independent random variables and $\mathrm{W}=\mathrm{X}+\mathrm{Y}$ obtain the pdf of W.

## OR

7.a) Write the properties of joint distribution function.
b) Prove that the variance of weighted sum of N random variables equals the weighted sum of all their covariances.
8. Define autocorrelation function of a random process. Write properties of auto correlation function of a WSS process and prove any three of them.
[10]

## OR

9.a) A random process $X(t)=A \cos \left(\omega_{0} t\right)+B \sin \left(\omega_{0} t\right)$ where $\omega_{0}$ is a constant and $A, B$ are uncorrelated zero mean random variables with same variances. Check whether $\mathrm{X}(\mathrm{t})$ is WSS or not?
b) Classify random processes and explain.
10. Derive the relationship between cross-power spectrum and cross-correlation function.

OR
11.a) The autocorrelation function of a random process $\mathrm{R}_{\mathrm{XX}}(\tau)=4 \cos \left(\omega_{0} \tau\right)$, where $\omega_{o}$ is a constant. Obtain its power spectral density.
b) Obtain the average power in the random process $X(t)=A \cos \left(\omega_{0} t+\theta\right)$ where $A, \omega_{o}$ are real constants and $\theta$ is a random variable uniformly distributed in the range $(0,2 \pi)$.
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JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, November/December - 2016 PROBABILITY THEORY AND STOCHASTIC PROCESSES (Common to ECE, ETM)
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## PART- A

(25 Marks)
1.a) A discrete random variable can be defined on a continuous sample space. State whether it is true or false. Give an example to support your claim.
b) Write the conditions to be satisfied by a function to be a random variable. [3]
c) Write the properties of probability density function.
d) Determine whether the following function is a valid probability distribution function or not? Write the properties used. $\quad G_{X}(x)=\frac{x}{a}[u(x-a)-u(x-2 a)]$.[3]
e) Write two properties of joint distribution function of random variables. [2]
f) State Central limit theorem.
g) Give an example of a deterministic random process.
h) Auto correlation function of a stationary random process is $R_{X X}(\tau)=25+\frac{4}{1+6 \tau^{2}}$. Find its variance.
i) Check whether the function below is a valid power density spectrum or not. $\frac{\omega}{j \omega^{6}+\omega^{2}+3}$.
j) Autocorrelation function of a random process is given by $R_{x X}(\tau)=3 \delta(\tau)$. Find and sketch its power density spectrum.

## PART-B

(50 Marks)
2.a) State and prove Bayes Theorem.
b) Define the terms outcome, event, sample space, mutually exclusive events. Consider the experiment of rolling of two fair dice simultaneously and represent its sample space. Also give examples of terms mentioned above related to this experiment.
[5+5]

## OR

3.a) Discuss the relative frequency approach and axiomatic approach of probability.
b) In a box there are 100 resistors whose resistances and tolerances are as shown in the table below. Let A be the event of drawing a $47 \Omega$ resistor, B be the event of drawing a resistor with $5 \%$ tolerance, and C be the event of drawing a $100 \Omega$ resistor. Find $\mathrm{P}(\mathrm{A} / \mathrm{B}), \mathrm{P}(\mathrm{A} / \mathrm{C})$ and $\mathrm{P}(\mathrm{B} / \mathrm{C})$.

|  | Tolerance |  |  |
| :---: | :---: | :---: | :---: |
| Resistance $(\Omega)$ | $5 \%$ | $10 \%$ | Total |
| 22 | 10 | 14 | 24 |
| 47 | 28 | 16 | 44 |
| 100 | 24 | 8 | 32 |
| Total | 62 | 38 | 100 |

4.a) Find the mean of Binomial random variable.
b) In a sports event javelin throw distances are well approximated by a Gaussian distribution for which mean is 30 m and standard deviation is 5 m . In a qualifying round, contestants must throw farther than 27 m to qualify. In the main event the record throw is 44 m .
i) What is the probability of being disqualified in the first round?
ii) In the main event what is the probability the record will be broken?

OR
5.a) Obtain the characteristic function of Poisson random variable.
b) X and Y are two statistically independent random variables related to W as $\mathrm{W}=\mathrm{X}+\mathrm{Y}$. Obtain the probability density function of Y in terms of probability density functions of X and Y .
6.a) Obtain the expression for conditional density $f_{X}(X / B)$ where event $B$ is defined as $\left\{\mathrm{y}_{\mathrm{a}} \leq \mathrm{Y} \leq \mathrm{y}_{\mathrm{b}}\right\}$.
b) Write short notes on jointly Gaussian random variables.

## OR

7.a) Two random variables X and Y have joint characteristic function $\varphi_{\mathrm{XY}}\left(\omega_{1}, \omega_{2}\right)=\exp \left(-2 \omega_{1}{ }^{2}-8 \omega_{2}{ }^{2}\right)$. Show that X and Y are uncorrelated zero mean random variables.
b) Two statistically independent random variables X and Y have mean values $\mathrm{E}[\mathrm{X}]=2$ and $\mathrm{E}[\mathrm{Y}]=4$. They have second moments $\mathrm{E}\left[\mathrm{X}^{2}\right]=8$ and $\mathrm{E}\left[\mathrm{Y}^{2}\right]=25$. Find Variance of $\mathrm{W}=3 \mathrm{X}-\mathrm{Y}$.
[5+5]
8.a) A random process is defined as $\mathrm{X}(\mathrm{t})=\mathrm{ACos}\left(\omega_{0} \mathrm{t}+\boldsymbol{\Theta}\right)$, where $\Theta$ is a uniformly distributed random variable in the interval $(0, \pi / 2)$. Check for its wide sense stationarity? A and $\omega_{o}$ are constants.
b) Classify random processes and explain.

OR
9.a) Define autocorrelation function of a random process. Write its properties and prove any two of them.
b) Explain the concept of time average and ergodicity. Write the conditions for a random process to be ergodic in mean and autocorrelation.
10.a) Derive the expression for power density spectrum of a random process.
b) Write the properties of power spectral density.

OR
11.a) Prove $\mathrm{S}_{\mathrm{YY}}(\omega)=\left.\right|_{H(\omega)^{2}} \mid \mathrm{S}_{\mathrm{XX}}(\omega)$. Where $\mathrm{X}(\mathrm{t})$ is input random process of an LTI system and $\mathrm{Y}(\mathrm{t})$ its output. $|H(\omega)|$ is the transfer function of the LTI system.
b) Define cross power density spectrum and write its properties.

## Code No: 123BT

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
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## PART- A

(25 Marks)
1.a) A box contains nine cards numbered through 1 to 9 , and $B$ contains five cards numbered through 1 to 5 . If a box is chosen at random, and a card is drawn which even numbered, what is the probability for the card to be from box A.
b) Let a die be weighted such that the probability of getting numbers from 2 to 6 is that number of times of probability of getting a1. When the die thrown, what is the probability of getting an even or prime number occurs.
c) Find the CDF of a random variable X , uniform over $(-3,3)$.
d) The density of a random variable X is given as $\mathrm{f}(\mathrm{x})=\mathrm{K}[\mathrm{U}(\mathrm{x})-\mathrm{U}(\mathrm{x}-4)]+0.25 \delta(\mathrm{x}-2)$. Find the probability of $\mathrm{X} \leq 3$.
e) X and Y are discrete random variables and their joint occurrence is given as

| $\mathrm{X} \backslash \mathrm{Y}$ | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- |
| 1 | $1 / 18$ | $1 / 9$ | $1 / 6$ |
| 2 | $1 / 9$ | $1 / 18$ | $1 / 9$ |
| 3 | $1 / 6$ | $1 / 6$ | $1 / 18$ |

Find the Conditional Mean of X , given $\mathrm{Y}=2$.
f) X and Y are two uncorrelated random variables with same variance. If the random variables $\mathrm{U}=\mathrm{X}+\mathrm{kY}$ and $\mathrm{V}=\mathrm{X}+(\sigma \mathrm{x} / \sigma \mathrm{y}) \mathrm{Y}$ are uncorrelated, find K .
g) State and prove the Periodicity Property of Auto Correlation function of a Stationary Random Process.
h) If $\mathrm{X}(\mathrm{t})$ is a Gaussian Random Process with a mean 2 and $\exp (-0.2|\tau|)$. Find the Probability of $\mathrm{X}(1) \leq 1$.
i) Verify that the cross spectral density of two uncorrelated stationary random processes is an impulse function.
j) The output of a filter is given by $\mathrm{Y}(\mathrm{t})=\mathrm{X}(\mathrm{t}+\mathrm{T})+\mathrm{X}(\mathrm{t}-\mathrm{T})$, where $\mathrm{X}(\mathrm{t})$ is a WSS process, power spectral density $S_{x x}(w)$, and $T$ is a constant. Find the power spectrum of $\mathrm{Y}(\mathrm{t})$.

## PART-B

(50 Marks)
2.a) Consider the experiment of tossing two dice simultaneously. If $X$ denotes the sum of two faces, find the probability for $\mathrm{X} \leq 6$.
b) A fair coin is tossed 4 times. Find the probability for the longest string of heads appearing to be three as a result of the above experiment.
c) In certain college, $25 \%$ of the boys and $10 \%$ of the girls are studying
 at random and studying mathematics, determine the probability that the student is a girl.
3.a) Coin A has a probability of head $=1 / 4$ and coin $B$ is a fair coin. Each coin is flipped four times. If X is the number of heads resulting from coin and Y denotes the same from coin B , what is the probability for $\mathrm{X}=\mathrm{Y}$ ?
b) A dice is thrown 6 times. Find the probability that a face 3 will occur at least two times.
4.a) Find the Moment generating function of a uniform random variable distribute over (A, B) and find its first and second moments about origin, from the Moment generating function.
b) A random variable X has a mean of 10 and variance of 9 . Find the lower bound on the probability of $(5<X<15)$.
[5+5]

## OR

5.a) Find the Moment generating function of a random variable X with density function

$$
f(x)=\left\{\begin{array}{c}
x, \text { for } 0 \leq x \leq 1 \\
2-x, \text { for } 1 \leq x \leq 2 \\
0, \text { else } \text { where }
\end{array}\right\}
$$

b) If $X$ is a Gaussian random variable $N\left(m, \sigma^{2}\right)$, find the density of $Y=P X+Q$, where P and Q are constants.
6.a) If $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3},----\mathrm{X}_{\mathrm{n}}$ are ' n ' number of independent and Identically distributed random variables, such that $X_{k}=1$ with a probability $1 / 2 ;=-1$ with a probability $1 / 2$. Find the Characteristic Function of the random Variable $Y=X_{1}+X_{2}+X_{3}+--$ $+X_{n}$.
b) If Independent Random Variables X and Y both of zero mean, have variance 20 and 8 respectively, find the correlation coefficient between the random Variables $\mathrm{X}+\mathrm{Y}$ and $\mathrm{X}-\mathrm{Y}$.
[5+5]

## OR

7.a) Let $\mathrm{X}=\operatorname{Cos} \theta$ and $\mathrm{Y}=\operatorname{Sin} \theta$, be two random variables, where $\theta$ is also a uniform random variable over $(0,2 \pi)$. Show that X and Y are uncorrelated and not independent.
b) If X is a random variable with mean 3 and variance 2, verify that the random Variables ' X ' and $\mathrm{Y}=-6 \mathrm{X}+22$ are orthogonal.
8.a) $\mathrm{X}(\mathrm{t})$ is a random process with mean $=3$ and Autocorrelation function $\mathrm{R}_{\mathrm{xx}}(\tau)=10 \cdot[\exp (-0.3|\tau|)+2]$. Find the second central Moment of the random variable $\mathrm{Y}=\mathrm{X}(3)-\mathrm{X}(5)$.
b) $\mathrm{X}(\mathrm{t})=2 \mathrm{ACos}(\mathrm{Wct}+2 \theta)$ is a random Process, where ' $\theta$ ' is a uniform random variable, over $(0,2 \pi)$. Check the process for mean ergodicity.

## OR

9.a) A Random Process $\mathrm{X}(\mathrm{t})=\mathrm{A} \cdot \operatorname{Cos}(2 \pi \mathrm{fc} \mathrm{t})$, where A is a Gaussian Random Variable with zero mean and unity variance, is applied to an ideal integrator, that integrates with respect to ' $t$ ', over $(0, t)$. Check the output of the integrator for stationarity.
 Variable with $\mathrm{p}(\mathrm{Y}=0)=\mathrm{p}(\mathrm{Y}=\pi)=1 / 2$. Find the mean and Variance of the Random Variable X(2).
10.a) Find and plot the Autocorrelation function of (i) Wide band white noise (ii) Band Pass White noise.
b) Derive the expression for the Cross Spectral Density of the input Process $\mathrm{X}(\mathrm{t})$ and the output process $\mathrm{Y}(\mathrm{t})$ of an LTI system in terms of its Transfer function.

OR
11.a) Compare and contrast Auto and cross correlations.
b) If $\mathrm{Y}(\mathrm{t})=\mathrm{A} \cdot \operatorname{Cos}\left(\mathrm{w}_{0} \mathrm{t}+\theta\right)+\mathrm{N}(\mathrm{t})$, where ' $\theta$ ' is a uniform random variable over $(-\pi, \pi)$, and $N(t)$ is a band limited Gaussian white noise process with PSD $=K / 2$. If ' $\theta$ ' and $\mathrm{N}(\mathrm{t})$ are independent, find the PSD of $\mathrm{Y}(\mathrm{t})$.
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